~~DCT~~

10.0 [illegible]

NATIONAL BUREAU OF STANDARDS REPORT

4854

THEORETICAL HEAT TRANSFER SCALING RELATIONSHIPS FOR FIRE ENDURANCE OF REINFORCED CONCRETE

by

A. F. Robertson and S. M. Genensky

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

U. S. DEPARTMENT OF COMMERCE
Sinclair Weeks, *Secretary*

NATIONAL BUREAU OF STANDARDS
A. V. Astin, *Director*

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its headquarters in Washington, D. C., and its major field laboratories in Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside back cover of this report.

WASHINGTON, D. C.

Electricity and Electronics. Resistance and Reactance. Electron Tubes. Electrical Instruments. Magnetic Measurements. Process Technology. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

Heat and Power. Temperature Measurements. Thermodynamics. Cryogenic Physics. Engines and Lubrication. Engine Fuels.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Nuclear Physics. Radioactivity. X-rays. Betatron. Nucleonic Instrumentation. Radiological Equipment. AEC Radiation Instruments.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Gas Chemistry. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

Mechanics. Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Organic Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion.

Mineral Products. Ceramic Engineering. Porcelain and Pottery. Glass. Refractories. Enameled Metals. Concreting Materials. Constitution and Microstructure.

Building Technology. Structural Engineering. Fire Protection. Heating and Air Conditioning. Floor, Roof, and Wall Coverings. Codes and Specifications.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. Components and Techniques. Digital Circuitry. Digital Systems. Analogue Systems. Applications Engineering.

- Office of Basic Instrumentation
- Office of Weights and Measures

BOULDER, COLORADO

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services.

Radio Propagation Engineering. Frequency Utilization Research. Tropospheric Propagation Research.

Radio Standards. High Frequency Standards Branch: High Frequency Electrical Standards. Radio Broadcast Service. High Frequency Impedance Standards. Microwave Standards Branch: Extreme High Frequency and Noise. Microwave Frequency and Spectroscopy. Microwave Circuit Standards.

NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT **NBS REPORT**

1002-12-4872 September 21, 1956 4854

THEORETICAL HEAT TRANSFER SCALING RELATIONSHIPS

FOR

FIRE ENDURANCE OF REINFORCED CONCRETE

by

A. F. Robertson and S. M. Genensky

The work reported was initiated on National Bureau of Standards' funds and further financial support was furnished by

Office of Chief of Engineers,
Department of Army

Directorate of Construction,
U. S. Air Force

NBS

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THEORETICAL HEAT TRANSFER SCALING RELATIONSHIPS FOR FIRE ENDURANCE OF REINFORCED CONCRETE

ABSTRACT

An attempt has been made to achieve a better appreciation of the relative fire endurance performance of various reinforced concrete structures. This has been done by solution in an approximate manner of the heat flow in a two dimensional model. Thermal constants for concrete were selected on the basis of providing a reasonable fit with limited experimental data.

The results are expressed in terms of the time required to reach temperatures of 750°F and 1000°F at various points within the model. These times are presented in the form of charts which are suggested for their possible use in estimating relative fire endurance of two structures which differ in one material or design parameter but are otherwise similar.

1. INTRODUCTION

Experimental determination of the fire resistance (1) performance of a structure is an expensive and time consuming procedure. It requires the construction of a large, often full size, specimen representative of a portion of a building. This structure is mounted either within or as a portion of the confining walls of a furnace. Determination of the fire performance of the specimen is defined as the period of time throughout which the specimen remains structurally sound and restricts heat transfer to a degree such that the temperatures of its unexposed portions remain below certain limits. Such determinations continue to be necessary because of difficulty in both expressing and analyzing the many factors which determine the ability of a structure to resist attack by fire. Even if it were possible to express all the complex structural and restraint details in precise technical fashion, it would still be extremely difficult to accurately solve the resulting heat flow and mechanics problem. Even if mechanical problems were neglected exact solution of the thermal problem would be difficult, because of the many material components, uncertainty as to the magnitude of the surface-to-gas-film heat transfer coefficients, and the complex manner of moisture migration in the specimen during test.

In spite of these difficulties it seemed desirable to attempt to develop some theoretical basis for predicting the thermal behavior of specimens during fire tests. Initial studies of the unsteady heat flow equations with the associated boundry conditions indicated that a simple scaling relationship should not be expected to exist. It appeared therefore that a relationship would only be developed as a result of systematic experimental study of a homologous series of specimens, or as a result of a number of numerical calculations of heat transfer in simple specimens. Menzel's data (2) seems to comprise one of the most carefully planned and executed experimental studies on the fire performance of simple slabs but the data are quite limited. Because of the time and costs involved in planning and performing a series of fire tests such as this, it appeared desirable to initiate the work by the alternative procedure, mathematical means.

This report therefore presents the results of a series of numerical computations on transient heat flow through simple structures.

2. ASSUMPTIONS

In order to facilitate solution of the problem, it appeared desirable to make the following assumptions:

1. Heat transfer within the solid occurred by conduction alone.

2. The length of the specimens were great compared with lateral dimensions and in the case of simple slabs both length and breadth were considered large compared with thickness.

3. The fire exposed surface of the specimen experienced a temperature-time relationship as defined by the standard curve for fire resistance tests described in ASTM method of test E-119. (1)

4. The solids involved were homogeneous and inert.

5. The mechanical behavior and thermal properties of the structure were independent of the temperature to which it was exposed until some limiting temperature had been reached.

6. Heat loss from the unexposed surface took place at a rate proportional to the temperature difference between the surface and the ambient at 68°F.

The first and second assumptions are considered justified for the range of materials and specimen sizes considered. Assumption three is considered desirable for the sake of simplicity and lack of detailed information on the magnitude of the heat transfer coefficients involved. It seems justified to a certain extent by the fact that furnace temperature is monitored by means of thermocouples enclosed within iron pipes.

The fourth assumption, that of a homogeneous and inert solid, presents a considerable simplification of the true condition. It seemed necessary however to permit practical solution of the problem with available computer facilities and time. This assumption requires neglecting the effects of absorbed moisture and the thermal discontinuities produced by reinforcing steel necessary in concrete construction. The first of these, moisture, is an important factor in determining the fire resistance behavior of many materials. The need to neglect the inclusion of its behavior in the calculations limits the usefulness of the results and requires that they serve only as indications of relative behavior of similar structures. The problem of the effects of reinforcing steel on heat flow within the structure was considered as a desirable but not essential refinement of the proposed method of analysis.

Assumption five was a necessary one. There is at present no known way of predicting the effects of high temperatures on the mechanical integrity of materials such as concrete and plasters. The effects of temperature on thermal properties of materials are probably better known. However neglect of the latter effect seems justified for the sake of simplicity in view of the other more restrictive assumptions which were found necessary.

The assumption that heat losses from the unexposed surface could be considered as Newtonian in character was considered necessary because of limitations in the storage system of the computer. It does not appear to limit the usefulness of the data developed for temperatures near the exposed surface of the specimens or the bottom of deep beams. For the case of very shallow beams and simple slabs the assumption is not well justified and does limit usefulness of the data on temperatures within the slab when unexposed surface temperatures become elevated.

These assumptions must therefore be considered as restricting the applicability of the data which have been developed. They should be considered in any use to which the data are put.

3. STATEMENT OF THE PROBLEM

A "T" beam formed the prototype of the construction considered for analysis. Figure 1 presents the geometry of the cross section of this beam. The upper plane surface represents the unexposed floor. Heat loss was assumed to take place from this surface according to Newton's law of cooling*. The lower surface of the slab and beam together with the two sides of the beam were assumed to be exposed to the furnace fires. These surfaces were made to follow the time temperature relationship as defined for fire resistance tests (1). The two extreme ends of the slab were assumed to be perfectly insulated.

These conditions may be expressed in mathematical symbolic form as follows:

$$\frac{\partial\theta}{\partial t} = \alpha\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right)$$

$$\left\{\begin{array}{l} -\infty < y < \infty, \quad 0 < x \leq a, \quad t > 0 \\ -b < y < b, \quad a \leq x < a + d, \quad t > 0 \end{array}\right\} \qquad (1)$$

with the following boundry equations and conditions:

$$\theta = f(t) \quad \left.\begin{array}{ll} -\infty < y \leq -b, \ x = a & y = -b, \ a \leq x \leq a + d \\ b \leq y < +\infty, \ x = a & y = b, \ a \leq x \leq a + d \\ -b \leq y \leq b, \ x = d + a & \end{array}\right\} t \geq 0 \qquad (2)$$

$$k\frac{\partial\theta}{\partial x} = -h\theta \quad -\infty < y < \infty, \ x = 0, \ t \geq 0 \qquad (3)$$

$$\left.\begin{array}{l} \frac{\partial\theta}{\partial y} \to 0 \text{ as } y \to +\infty \\ \qquad\quad \text{ as } y \to -\infty \end{array}\right\} 0 \leq x \leq a, \ t \geq 0 \qquad (4)$$

$$\theta = 0 \quad -\infty < y < \infty, \ 0 \leq x \leq a, \ t = 0 \qquad (5)$$

$$-b \leq y \leq b, \ a \leq x \leq a + d, \ t = 0$$

$$\frac{\partial\theta}{\partial y} = 0 \quad 0 \leq x \leq a + d, \ y = 0, \ t \geq 0 \qquad (6)$$

---

*The heat transfer coefficient entering this equation was altered to account for heat loss by radiation.

where

x and y are rectangular coordinates in the plane of heat flow.

t is the time measured from some initial time t = 0.

$\theta = \theta(x,y,t)$ is the temperature a function of two rectangular coordinates x and y and also the time t.

k is the thermal conductivity of the material forming the beam.

ρ is the density of the material forming the beam.

c is the specific heat of the material forming the beam.

$\alpha = k/\rho c$ is the diffusivity of the material forming the beam.

h is the heat transfer coefficient between the slab material and air.

f(t) is the temperature of the exposed face of the T-beam and depends upon the time t.

Although common engineering units were used for the present study it is obvious that any consistent set of units may be used.

Since in practice condition (4) is satisfied if the slab extends laterally about six times its thickness to the right and left of vertical faces of the beam, that condition may be relaxed to read:

$$\frac{\partial\theta}{\partial y} = 0 \quad y = \pm(\ell+b),\ 0 \leq x \leq a,\ t \geq 0 \qquad \ell \geq 6a \tag{4'}$$

and conditions (1), (2), (3) and (5) may be modified by replacing the symbol "∞" by "$\ell+b$". Let these modified equations be known as equations (1'), (2'), (3') and (5') respectively.

Consideration of figure 1 and equations (1')-(5') and (6) indicates that the problem is symmetric about the ordinate y = 0 and thus only the inverted L section to the right of this line need be considered.

Since a high speed digital machine such as SEAC is most useful for the elementary operations of addition, subtraction,

multiplication and division the equations (1')-(5') and (6) must be so approximated that only these arithmetic operations are involved. The methods of finite differences afford sufficient approximating relationships.

Let $T^p_{i,j} = \Theta(i\Delta x, j\Delta y, p\Delta t)$ where i, j and p may be non-negative integers.

Then $$\frac{\partial \Theta}{\partial t} \approx \frac{T^{p+1}_{i,j} - T^p_{i,j}}{\Delta t} \qquad (7)$$

$$\frac{\partial \Theta}{\partial x} \approx \frac{T^p_{i+1,j} - T^p_{i,j}}{\Delta x} \qquad (8)$$

$$\frac{\partial \Theta}{\partial y} \approx \frac{T^p_{i,j+1} - T^p_{i,j}}{\Delta y} \qquad (9)$$

$$\frac{\partial^2 \Theta}{\partial x^2} \approx \frac{T^p_{i+1,j} + T^p_{i-1,j} - 2T^p_{i,j}}{(\Delta x)^2} \qquad (10)$$

$$\frac{\partial^2 \Theta}{\partial y^2} \approx \frac{T^p_{i,j+1} + T^p_{i,j-1} - 2T^p_{i,j}}{(\Delta x)^2} \qquad (11)$$

Assuming $\Delta y = \Delta x$, using (6), (7), (10), and (11), letting $\lambda = \frac{\partial \Delta t}{(\Delta x)^2}$ and $\mu = 1 - 4\lambda$ and rearranging terms, (1') may be approximated by:

$$T^{p+1}_{i,j} \equiv \lambda\left[T^p_{i+1,j} + T^p_{i-1,j} + T^p_{i,j+1} + T^p_{i,j-1}\right] + \mu T^p_{i,j}$$

$i = 1,2,\ldots,m_1 \quad j = 1,2,\ldots,n_1-1 \quad p = 0,1,2,\ldots$

where $m_1 \Delta x = a$ and $n_1 \Delta y = \ell + b$

and

$i = m_1, m_1+1,\ldots, m_2-1 \quad j = 1,2,\ldots,n_2-1 \quad p = 0,1,2,\ldots$

where $m_2 \Delta x = a + d \qquad n_2 \Delta y = b$ (12)

It is thus seen that the inverted L-section has been replaced by a rectangular array of points (see figure 2) and equation

(12) may be used to compute the temperature at any of the interior points at any time $(p + 1)\Delta t$ provided that the temperature at all the grid points is known at time $p\Delta t$.

In difference notation boundry condition (2') becomes:

$$T^p_{i,j} = f(p\Delta t) \quad \left.\begin{array}{ll} i = m_1 & j = n_2,\ n_2 + 1, \ldots, n_1 \\ i = m_2 & j = 0,1,2,\ldots, n_2 \\ i = m_1,\ m_1 + 1,\ldots, m_2 & j = n_2 \end{array}\right\} p = 0,1,2,\ldots \tag{13}$$

The values of $f(t)$ for $t = p\Delta t$ were tabulated before the problem was run and were stored on an auxiliary tape unit. In the experimental setup, the furnace temperature is caused to follow a curve adopted by the ASTM for use in fire tests (1).

Using equation (8), boundry condition (3') becomes after rearranging terms:

$$T^p_{o,j} = \frac{1}{\left(1 + \frac{h}{k}\Delta x\right)} T^p_{1,j} = \frac{kT^p_{1,j}}{k-h\Delta x} \tag{14}$$

$$j = 0,1,2,\ldots,n_1 \quad p = 1,2,\ldots$$

At the extremities of the slab $(y = b + \ell)$ heat flow is assumed normal to the y coordinate as required by boundry condition (4'). Equation 12 then simplifies to that for one dimensional heat flow:

$$T^{p+1}_{i,j} = \lambda(T^p_{i+1,j} + T^p_{i-1,j}) + \mu_1 T^p_{i,j} \tag{15}$$

where $\mu_1 = 1-2\lambda$ and

$$i = 1,2,\ldots,m_1-1 \quad j = n_1 \quad p = 0,1,2,\ldots$$

Boundry condition (6) is accounted for by a simple device. An extra line of points is placed below the abscissa $y = 0$ (i.e. along the line $y = -\Delta y$) and the points in this column are assigned the values, assumed or computed, for the points in the column $y = \Delta y$. Thus:

$$T^p_{i,1} = T^p_{i,-1} \quad i = 1,2,\ldots,m_2-1 \quad j = 1 \quad p = 1,2,\ldots \tag{17}$$

and temperatures along the centerline of the beam (j = 0) may be expressed as follows:

$$T^{p+1}_{i,j} = \lambda\left[T^p_{i+1,j} + T^p_{i-1,j} + T^p_{i,j+1} + T^p_{i,j-1}\right] + \mu T^p_{i,j} \tag{18}$$

where $i = 1,2,\ldots,m_2-1 \quad j = 0 \quad p = 1,2,\ldots$

Equations (12)-(18) were thus used in the preparation of a code for the numerical calculations instead of equations (1)-(6) or (1')-(5') and (6).

In using the digital computer for solution of this type of problem complete temperature data for all grid points were recorded at periodic intervals during the computation process. The computation process was terminated when the temperature at grid point $y = 0$ and $x = (m_2-n_2)\,\Delta x$ reached 1000°F. This point was selected because it was located on a plane through the vertical centerline of the beam and was equally remote from both sides and bottom surface of the beam and therefore represented the position within the lower portion of the beam at which reinforcement could be placed in such a manner to secure the maximum amount of "cover" or protection from fire. The temperature of 1000°F corresponds to that at which the elastic limit of the steel reinforcement becomes approximately equal to the working stresses customarily used in such construction.

4. PARAMETERS INVESTIGATED

Original plans for this study included the solution of the heat transfer problem for all combinations of three sets of thermal and physical properties representative of a wide range of concretes, five different beam designs as represented by the ratio of beam depth to slab thickness, (B/S ratio) and eight different beam sizes as determined by linear scale ratio of model to prototype. This program would require the solution and analysis of 120 problems. For purposes of conserving time only thirty-three of these problems have been solved, see Table I. These were selected to provide as comprehensive an investigation as practicable of the full range of variables involved.

5. THERMAL PROPERTIES

As mentioned earlier there was considerable uncertainty as to the values of the thermal properties of materials during performance of a fire resistance test. The problem was chiefly one of physical and chemical phase changes together with the migration behavior of gases and vapors. Water absorbed within the specimen was one important factor influencing uncertainties of these properties. Because of this, rather arbitrary methods were necessary in selecting the thermal properties to be used in this study. Data on dry concretes presented in reference 4 served as a guide in this selection. However, conductivity values were increased by about 25 percent to be more representative of conditions existing at elevated temperatures. The values selected were those for densities of about 140, 120, and 34 lb/ft^3 concrete as shown below for concrete A, B and D.

Assumed Thermal Properties of Concrete

Concrete	ρ	k	α'	α
	lb/ft^3	BTU/hr ft°F	ft^2/hr	ft^2/hr
A	140	(.90) .905	.0323	.0112
B	120	(.58) .580	.0242	.0084
C	80	(.28) .101*	.0149	.0056
D	34	(.12) .101	.0149	.0056

*As will be explained later this value of k though not exact may be used without serious error. It is not however consistent with the value of α tabulated.

In this table α' is the diffusivity based on a specific heat of 0.2 x BTU/lb°F. The values shown were used in preliminary trial solutions of the problem for simple slabs similar to those for which a few experimental results were available. It was found that the calculated fire endurance periods as indicated by temperature rise within the slab were considerably shorter than indicated by experiment. Because of this the values of α' were reduced by a factor

approximating 3 resulting in the values of α tabulated above. These values of α and k were then used in performing the calculations. These values are shown plotted in figure 3.

As indicated by equations (1) and (3) of the previous section, α is important in controlling the rate with which heat is distributed within the solid and k functions to control the rate of heat loss and thus the temperature of the unexposed surface of the specimen. For the case of relatively deep beams the temperature of reinforcement is almost independent of the conditions at the unexposed surface. The fact that the curve of thermal diffusivity as a function of density of concrete seems to have a minimum value at a density of about 60 lb/ft^3, permits the use of calculations for 34 lb ft^3 concrete as indicative of performance of concrete of 80 lb/ft^3. Thus under these limitations it is possible to produce data for four concrete densities by the use of only three sets of calculations.

6. CONVECTION AND RADIATION LOSSES

At the time this problem was prepared for solution by means of SEAC, limitations in storage capacity prevented the use of a temperature dependent heat transfer coefficient (h) at the unheated surface. It was necessary therefore to consider these losses as Newtonian for any particular problem. However a method was devised for estimating maximum temperature rise at the unexposed surface, a value for h was then selected as representative of the average effective value over that temperature range. Figure 4 presents these average values of surface heat transfer coefficients as a function of unexposed surface temperature rise at the end of the test. These coefficients were based on radiation and convection losses from the surface to the ambient. This ambient was assumed constant at 68°F and the emissivity of the specimen surface was assumed as unity. The convection losses were assumed to follow Jakob's relationship reference 4:

$$h_c = 0.275\ \theta^{1/3}$$

7. RESULTS AND DISCUSSION

A typical solution of the T beam problem is presented in figure 5. This shows temperatures within the specimen representative of concrete A and having a beam depth to slab thickness ratio of 4.2. This chart presents the data which were periodically "read out" of the digital computer. In

general such data were recorded from 6 to 15 times during the course of solution of each problem. It obviously was necessary to select another means of presentation of the data. The method used consisted in plotting temperature-time data for each of the encircled points near the lower corner of the beam. These particular points were considered of primary importance because they occur in positions frequently occupied by the tensile reinforcing steel. Fire endurance times were then read off these charts corresponding to the attainment of temperatures of 1000°F and 750°F for each of the points involved. These data together with similar data for beams of other scale sizes were replotted to form the family of curves shown in figures 6 and 7. Figures 8 and 9 present data on the variations of endurance caused by changing the beam to slab ratio. The data presented here are for concrete A, scale ratio of one, and critical temperatures θ_c of 1000°F and 750°F respectively. Figures 10 and 11 present similar data for the case of simple slabs as exemplified by the extremities of the top slab of the beam. Here heat flow is one dimensional and follows the relationship shown in equation (15) together with suitable boundry conditions. In deriving these charts a series of computations were made for each specimen for a range of unexposed surface temperature rise conditions. In this manner the limitations involved by the assumption of Newtonian heat flow at the unexposed surface were largely avoided.

Original plans called for preparation and presentation of similar charts for concrete of other thermal properties. However analysis of the computations indicated that a simpler method might be used. Figures 12, 13, and 14 together with figures 6, 7, 10 and 11 present a means for estimating fire endurance performance of specimens formed from concretes B, D, and less accurately, those of concrete C on the basis of results obtained with specimens fabricated with concrete A. Figure 12 was developed from a plot of calculated results for beams having scale ratios varying from 1/4 to 2 and beam to slab ratios varying from 1.8 to 7.4. From it the relative performance can be estimated for beams which differ only in thermal properties. Figures 13 and 14 were developed from figures 8 and 9 for the scale ratio of 1. Because of the generalization shown in figure 12, figures 13 and 14 may be applicable to conc etes of a wide range of thermal properties.

8. DISCUSSION

Little use would be served by lengthy speculation on the effects of the various assumptions which were considered necessary for solution of the problem. The present discussion will be confined to a review of the data which have been developed and brief comments on the manner in which they might be used.

The plot of isotherms within the beam shown in figure 5 is useful as an illustration of the temperature distribution within a given beam at a particular time. The vulnerability of exposed corners to thermal attack is evident. The selection of such corners for analysis of relative performance based on heat transfer seems well justified. Consideration of the triangular area A O B for the case of this and other beams of large beam to slab depth ratio indicates that the results obtained could be considered applicable to columns too. Likewise the portions of the floor slab remote from the beam are representative of conditions in a single slab.

As mentioned previously the data tabulated here are those for points along the diagonal O A of the beam. Figures 6 and 7 present such data in the form of fire endurance as a function of beam size or scale ratio. The latter has been selected as a parameter for presentation of the data in preference to use of an actual linear dimension in an effort to emphasize the relative rather than absolute character of the fire endurance data plotted. The two curves differ only in the temperature which has been assumed as the criterion of failure at the point in question. These correspond to 1000°F and 750°F respectively. The former is considered applicable to steel as used in conventionally reinforced concrete construction. The latter is presented as possibly of interest in connection with studies of behavior of prestressed concrete structures. Both curves are the result of calculations based on concrete A and a beam to slab depth ratio of 4.2. The data are however, considered applicable to the case of any T beams with values of this ratio greater than 3.5 provided the beam stem thickness approximates twice the slab thickness.

Each of these charts consists of a group of curves starting near the origin and diverging slightly as scale size increases. These individual lines correspond to the endurance to be expected when similar structures are tested each of which has the reinforcement located at a fixed ratio of the maximum possible depth. An additional family of curves has been developed in such a way that for any one of these the product of scale ratio and percentage depth is a constant. These curves then indicate the change in fire endurance with

scale ratio when a fixed amount of cover is provided for the steel. It will be observed that the fire endurance is reduced as the scale size is decreased resulting in placement of the reinforcement near the mid plane of the beam or 100% depth position. This effect is evidently a result of the reduction in the amount of material backing up the reinforcement and thus capable of soaking up heat. When increasing the scale ratio of the beam and maintaining the reinforcement at a fixed depth little improvement in fire endurance will result when the size is increased beyond the point at which the reinforcement is at less than 50 percent of maximum depth. In fact the data indicate that for very large scale ratios and resulting small depth ratios the endurance may again show a very slight decrease. This probably results from reduction of heat losses on the unexposed surface of the specimen.

Figures 8 and 9 present endurance data for T beams of a scale ratio one as a function of variation in beam to slab depth ratio. These curves are for critical temperatures of 1000°F and 750°F respectively. These again are for concrete A. The charts provide the justification for the previous statement that endurance is unaffected by increasing beam to slab depth ratio for values greater than about 3.5. In fact for shallow depth ratios of 10 to 20 percent the endurance does not seem to change greatly for any beam to slab depth ratio greater than two. The figures show the advantages to be gained by use of small beam to slab ratios. The increased fire endurance is a direct result of the usefulness of the concrete slab in dissipating heat that would otherwise tend to cause more rapid rise in steel temperatures.

Figure 12 presents a plot of performance of T beams differing only in thermal properties. The data for the less dense concretes are plotted against fire endurance performance of specimens of concrete A but otherwise identical in construction and failure criterion. Data plotted are based on the limited calculations performed on the two lower density materials and cover the following range of variables:

Critical temperature	1000°F and 750°F
Beam to slab ratios	1.8 to 7.4
Scale ratios	1/4 to 2
Depth ratios	0.0 to 1.0 or less as limited by endurance of 480 min.

This chart therefore provides a simple direct means for estimating relative performance of specimens differing only in thermal properties.

The good correlation shown in figure 12 between the fire endurance of specimens having differing thermal properties was unexpected. Because of this correlation a separate plot was prepared in which the slope of the plots shown in figure 12, very closely approximating the endurance ratio t_x/t_1, was plotted against the reciprocal of the ratio of the corresponding thermal diffusivities α_1/α_x. When this was done for each of the three values of α, it was found that a straight line relationship existed between the two ratios. While there was not a strict one to one relationship between the two parameters, it did indicate close approach to the theoretical relationship of:

$$t_x/t_1 = \alpha_1/\alpha_x$$

for the case of simple slabs and step temperature functions on the exposed surface.

Figures 13 and 14 have been derived from the charts of figures 8 and 9. They show fire endurance of specimens of the indicated beam depth to slab thickness ratios relative to that of similar specimens but having beam depth to slab thickness ratio of 4.2 or greater. They are applicable to beams of all thermal properties and a range of beam to slab depth ratios. These two charts were derived only for the scale ratio of one. However, analysis of the data available for concrete A indicates that little error will be introduced when they are used for the scale ratios varying from 1/4 to 2.

9. SUMMARY

The charts presented in this paper are based on theoretical calculations of unsteady heat transfer through homogeneous materials. It is believed that they should provide a useful means for estimating relative performance of simple slabs and T-beam specimens having a stem to slab thickness ratio of two but which differ otherwise in only one design or construction feature such as:

1. Depth of reinforcement
2. Scale size of specimen
3. Beam depth to slab thickness ratio
4. Thermal properties of material

The extent to which the calculated scaling relationships can be shown to be applicable to actual performance is expected to depend largely on the applicability of the assumptions to the problem in hand. Experimental work is planned to obtain data which will help evaluate the usefulness of the derived relationships.

Any application of these relationships to actual fire endurance problems should be made with caution and due respect for the assumptions on which the solutions presented have been based.

10. REFERENCES

1. "Standard Methods of Fire Tests of Building Construction and Materials", ASTM Designation E119-55, 1955, ASTM Standards Part 4, American Society for Testing Materials, Philadelphia, Pennsylvania.

2. "Tests of the Fire Resistance and Thermal Properties of Solid Concrete Slabs and Their Significance", C. A. Menzel, ASTM Proc. V 43, pp 1099-1153, 1943.

3. "Lightweight Aggregate Concrete", Housing and Home Finance Agency, 1949, U. S. Government Printing Office.

4. "Heat Transfer", Jakob, John Wiley & Sons, 1949.

Table I

Schedule of Beam Problems Solved

Size and Shape Parameters	Material														
	Concrete A					Concrete B					Concrete D				
Scale Ratio \ Beam to Slab Ratio B/S	1.8	2.2	2.6	4.2	7.4	1.8	2.2	2.6	4.2	7.4	1.8	2.2	2.6	4.2	7.4
1/4	x			x					x					x	
1/2	x	x	x	x		x					x				
3/4				x											
1	x	x	x	x	x	x			x		x		x	x	x
5/4				x											
3/2	x			x											
7/4				x											
2	x			x		x			x		x			x	
5				x											

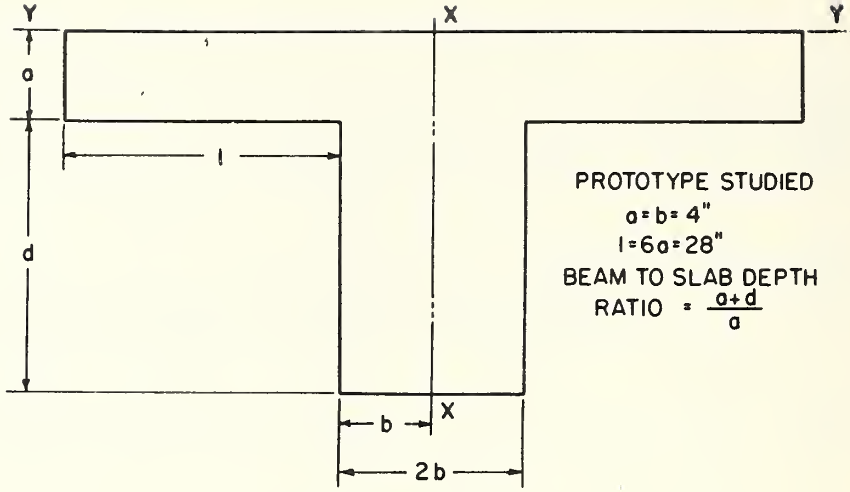

Y
X
Y
a
l
d
PROTOTYPE STUDIED
a = b = 4"
l = 6a = 28"
BEAM TO SLAB DEPTH
RATIO = (a+d)/a
X
b
2b

FIG. 1

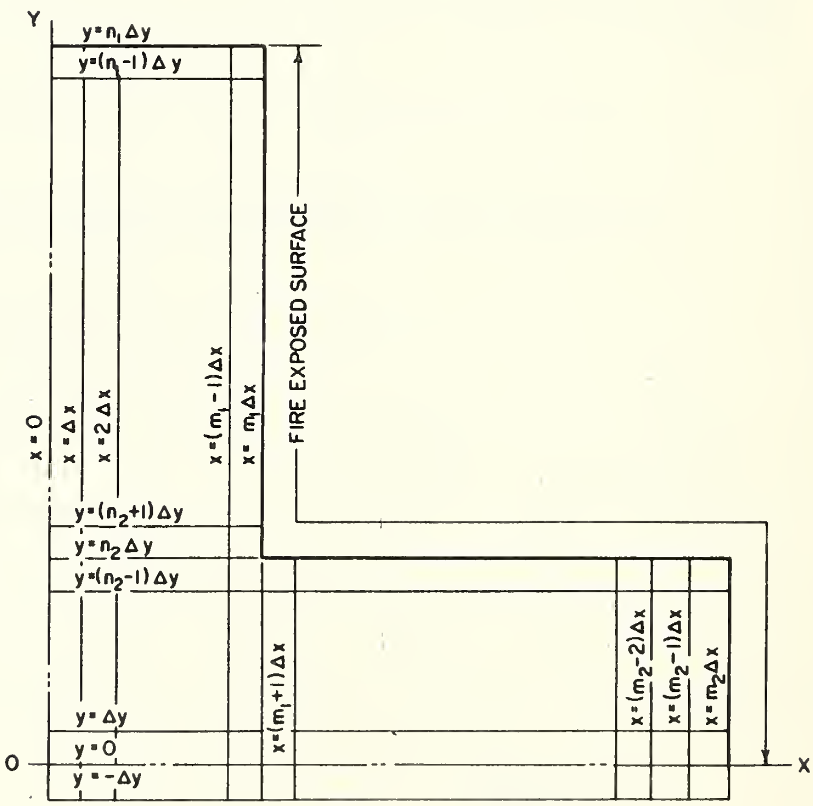

Y
y = n₁ Δy
y = (n₁ - 1) Δy
x = 0
x = Δx
x = 2 Δx
x = (m₁ - 1) Δx
x = m₁ Δx
FIRE EXPOSED SURFACE
y = (n₂ + 1) Δy
y = n₂ Δy
y = (n₂ - 1) Δy
x = (m₁ + 1) Δx
x = (m₂ - 2) Δx
x = (m₂ - 1) Δx
x = m₂ Δx
y = Δy
y = 0
y = -Δy
O
X

FIG. 2

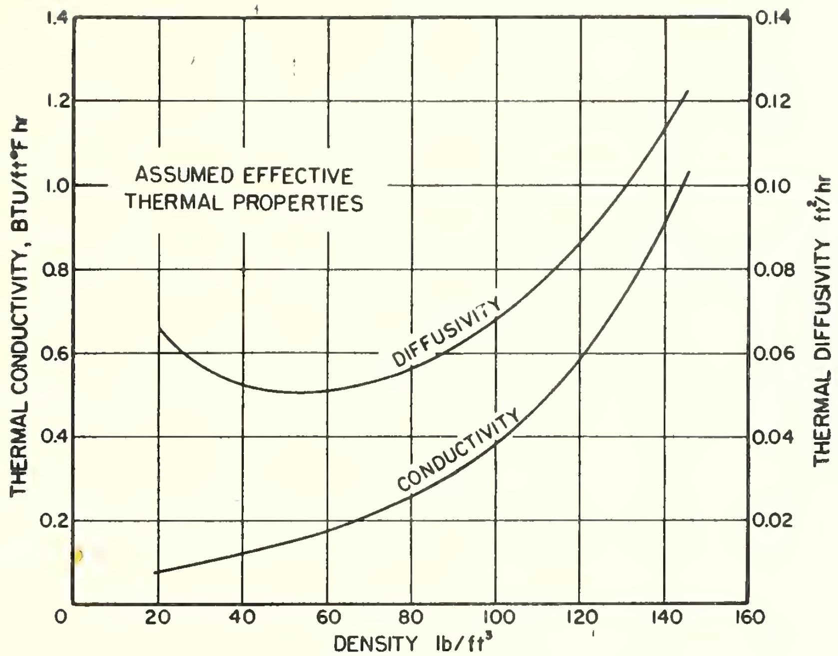

THERMAL CONDUCTIVITY, BTU/ft°F hr
THERMAL DIFFUSIVITY ft²/hr
ASSUMED EFFECTIVE
THERMAL PROPERTIES
DIFFUSIVITY
CONDUCTIVITY
DENSITY lb/ft³
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.14
0.12
0.10
0.08
0.06
0.04
0.02
20
40
60
80
100
120
140
160

FIG. 3

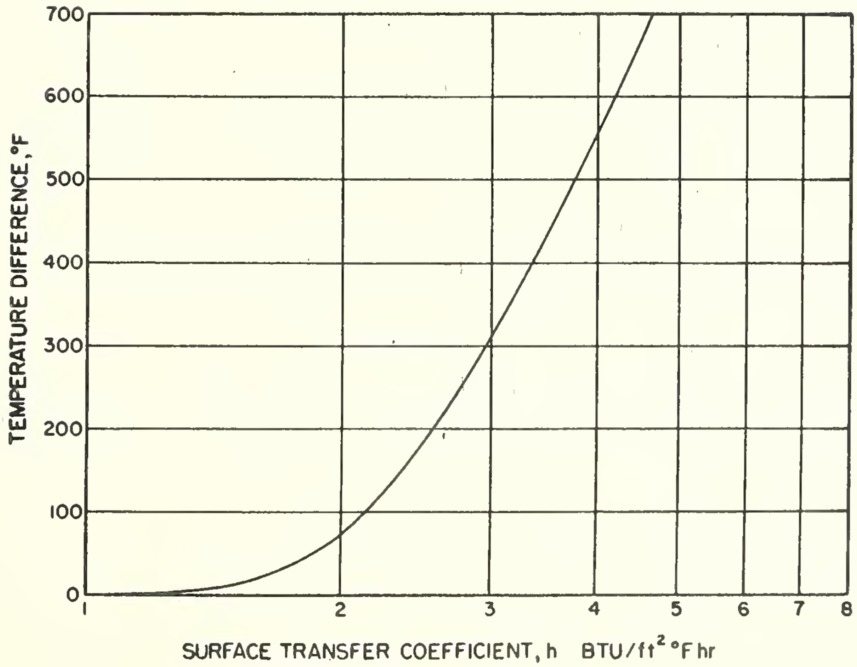

TEMPERATURE DIFFERENCE, °F
SURFACE TRANSFER COEFFICIENT, h BTU/ft² °F hr
700
600
500
400
300
200
100
0
1
2
3
4
5
6
7
8

FIG. 4

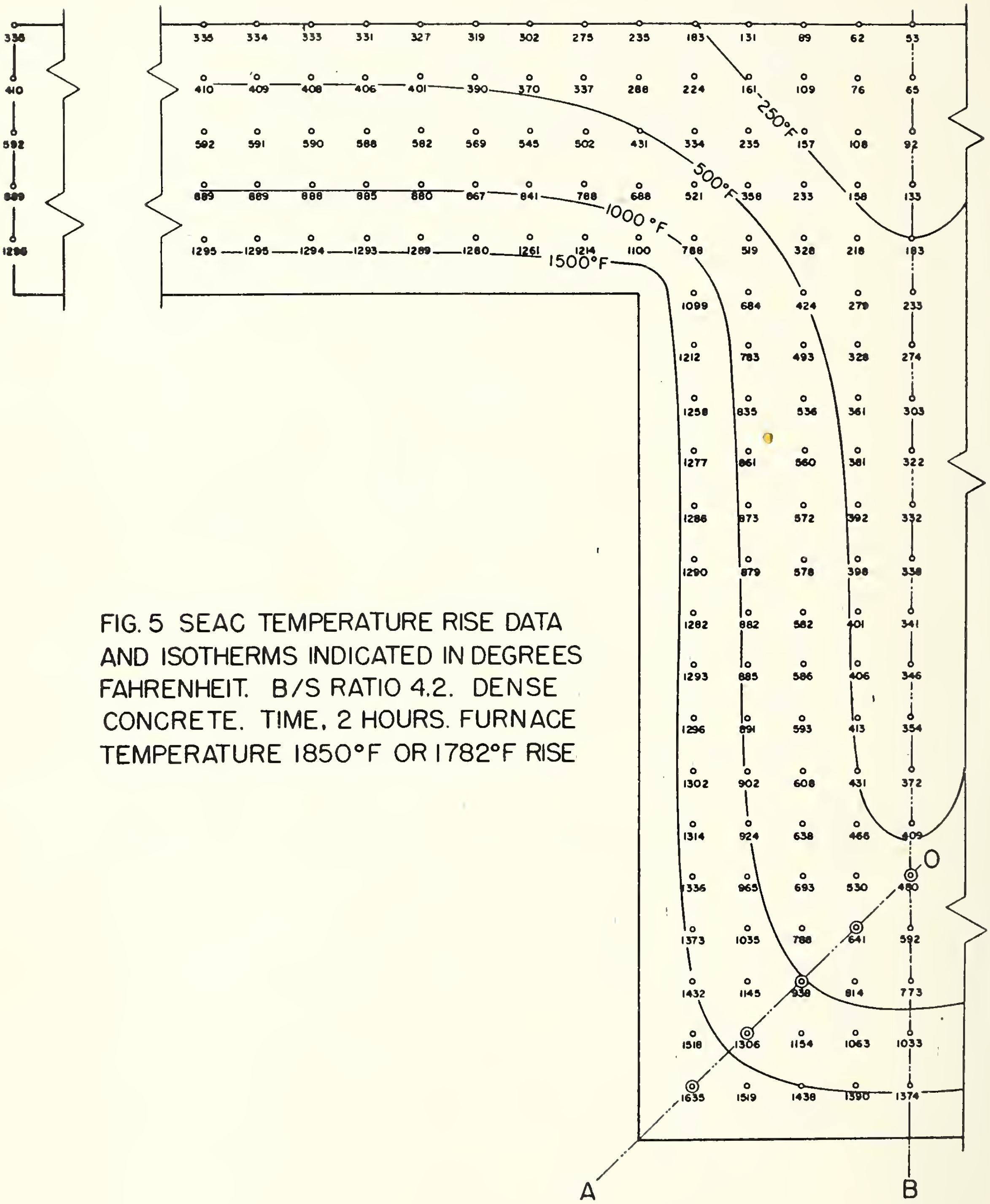
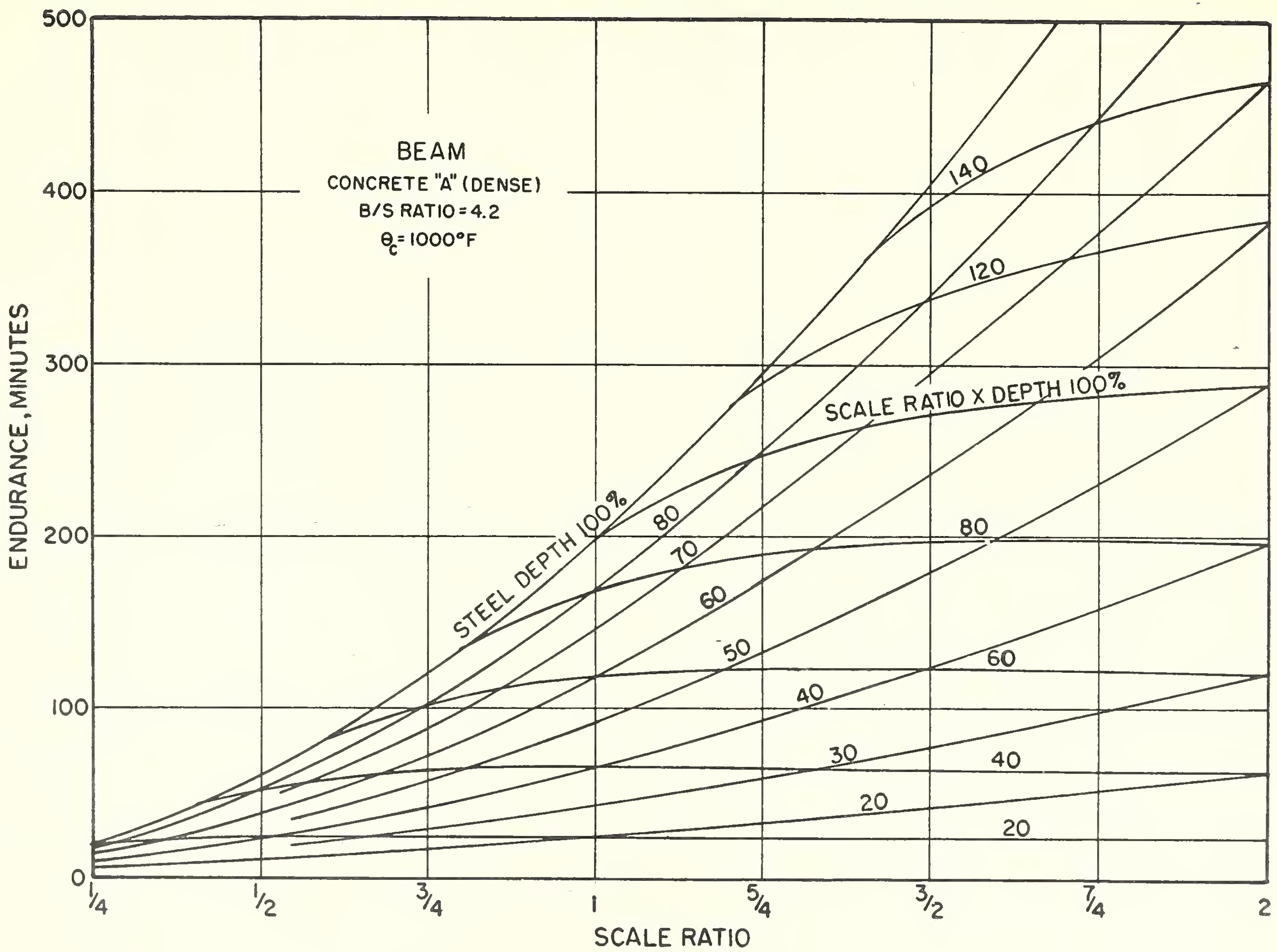


FIG. 5 SEAC TEMPERATURE RISE DATA AND ISOTHERMS INDICATED IN DEGREES FAHRENHEIT. B/S RATIO 4.2. DENSE CONCRETE. TIME, 2 HOURS. FURNACE TEMPERATURE 1850°F OR 1782°F RISE



BEAM
CONCRETE "A" (DENSE)
B/S RATIO = 4.2
θ_c = 1000°F
500
400
300
200
100
0
ENDURANCE, MINUTES
1/4
1/2
3/4
1
5/4
3/2
7/4
2
SCALE RATIO
STEEL DEPTH 100%
80
70
60
50
40
30
20
SCALE RATIO X DEPTH 100%
140
120
80
60
40
20

FIG. 6

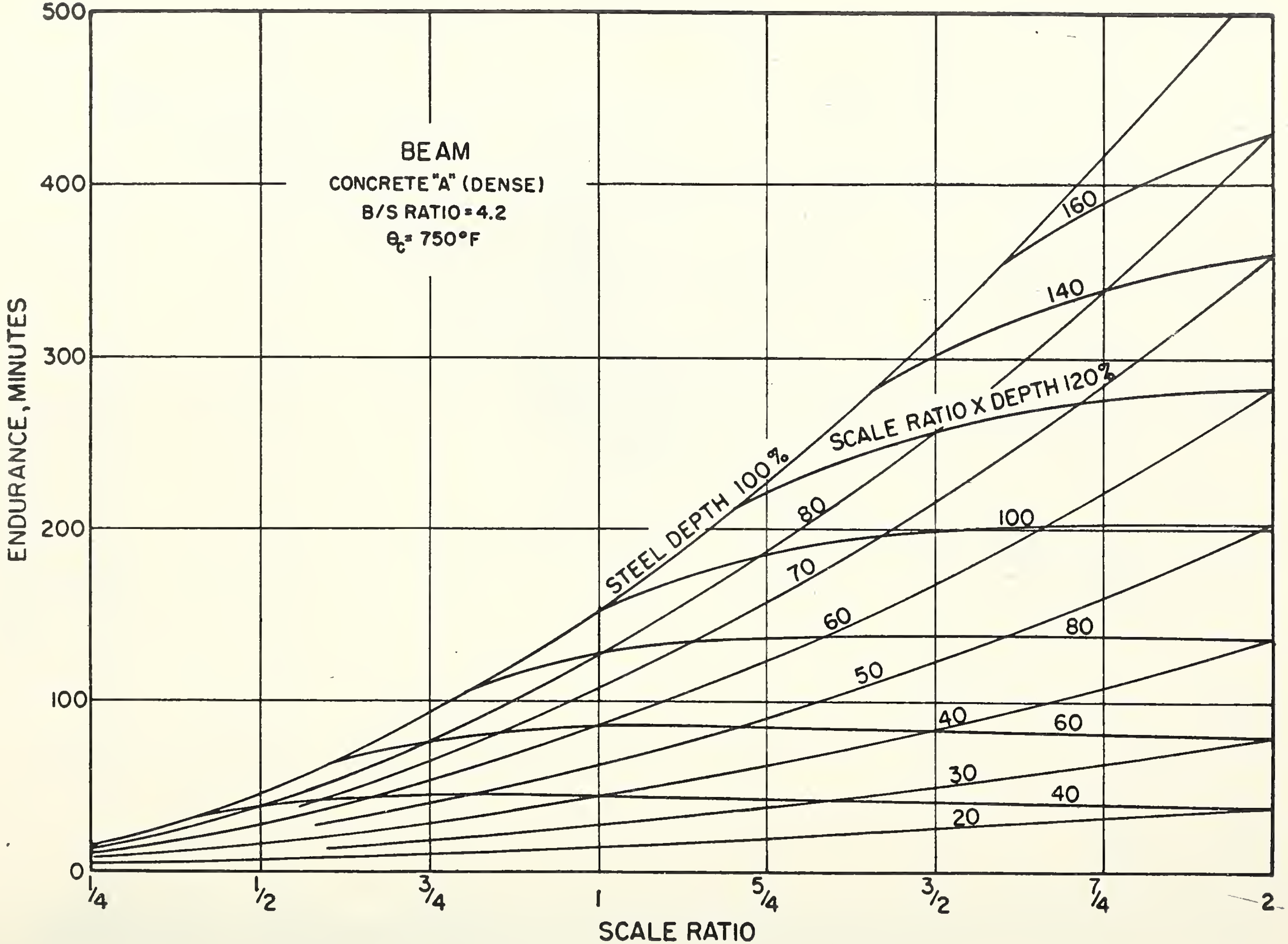
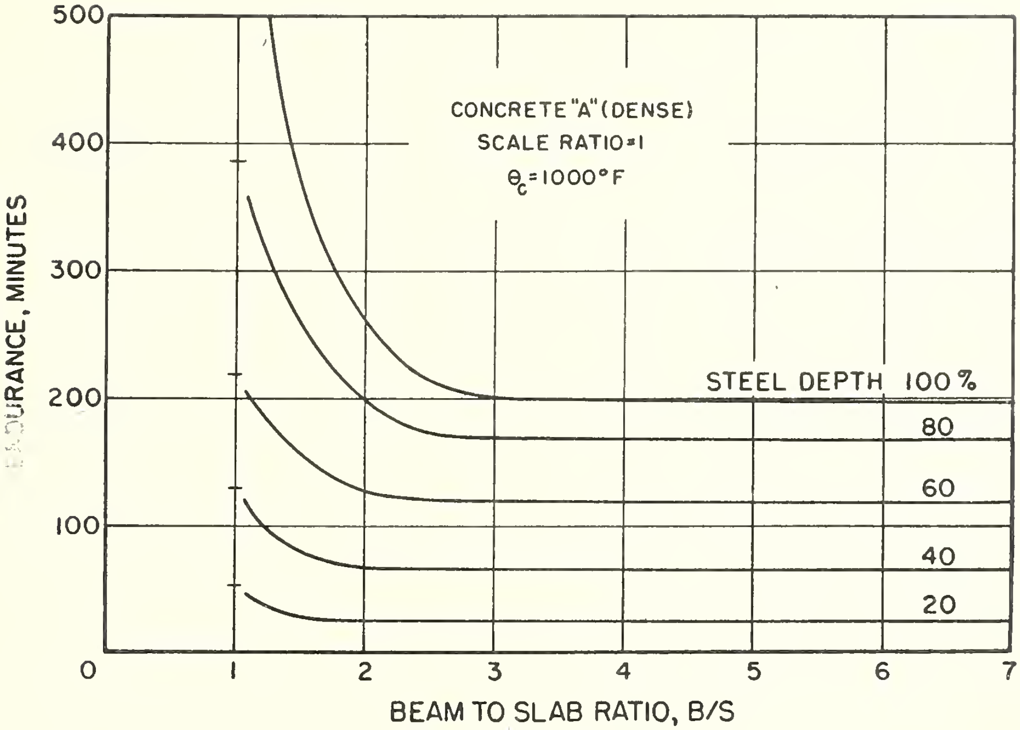
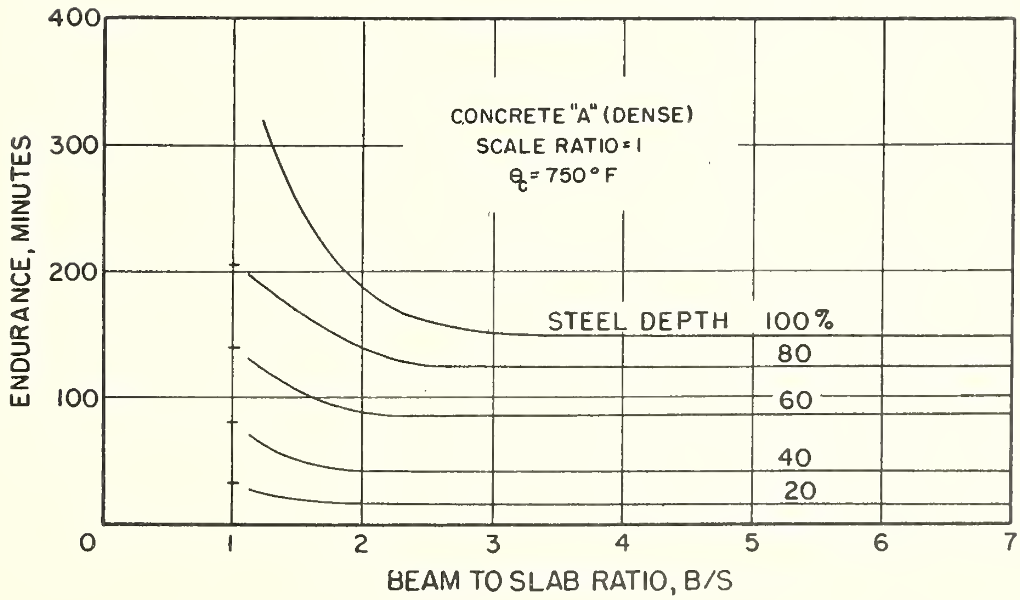

BEAM
CONCRETE "A" (DENSE)
B/S RATIO = 4.2
θ_c = 750°F
ENDURANCE, MINUTES
500
400
300
200
100
0
SCALE RATIO
1/4
1/2
3/4
1
5/4
3/2
7/4
2
STEEL DEPTH 100%
SCALE RATIO X DEPTH 120%
160
140
100
80
70
60
50
40
30
20
80
60
40

FIG. 7



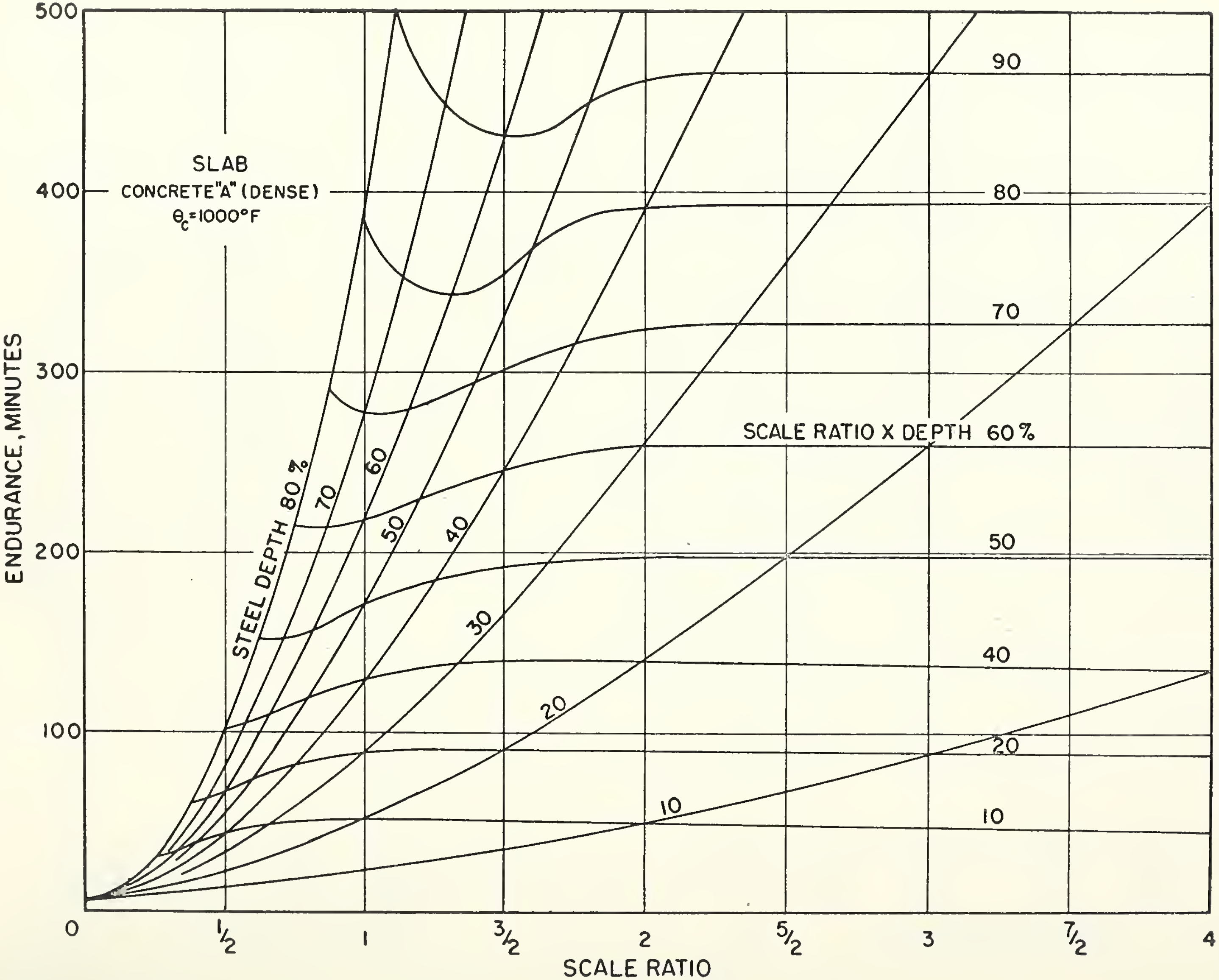
CONCRETE "A" (DENSE)
SCALE RATIO = 1
θ_c = 1000°F
STEEL DEPTH 100%
80
60
40
20
ENDURANCE, MINUTES
BEAM TO SLAB RATIO, B/S

FIG. 8



CONCRETE "A" (DENSE)
SCALE RATIO = 1
θ_c = 750°F
STEEL DEPTH 100%
80
60
40
20
ENDURANCE, MINUTES
BEAM TO SLAB RATIO, B/S

FIG. 9



SLAB
CONCRETE "A" (DENSE)
θ_c=1000°F
ENDURANCE, MINUTES
500
400
300
200
100
0
½
1
3/2
2
5/2
3
7/2
4
SCALE RATIO
STEEL DEPTH 80 %
70
60
50
40
30
20
10
SCALE RATIO X DEPTH 60 %
90
80
70
50
40
20
10

FIG.10

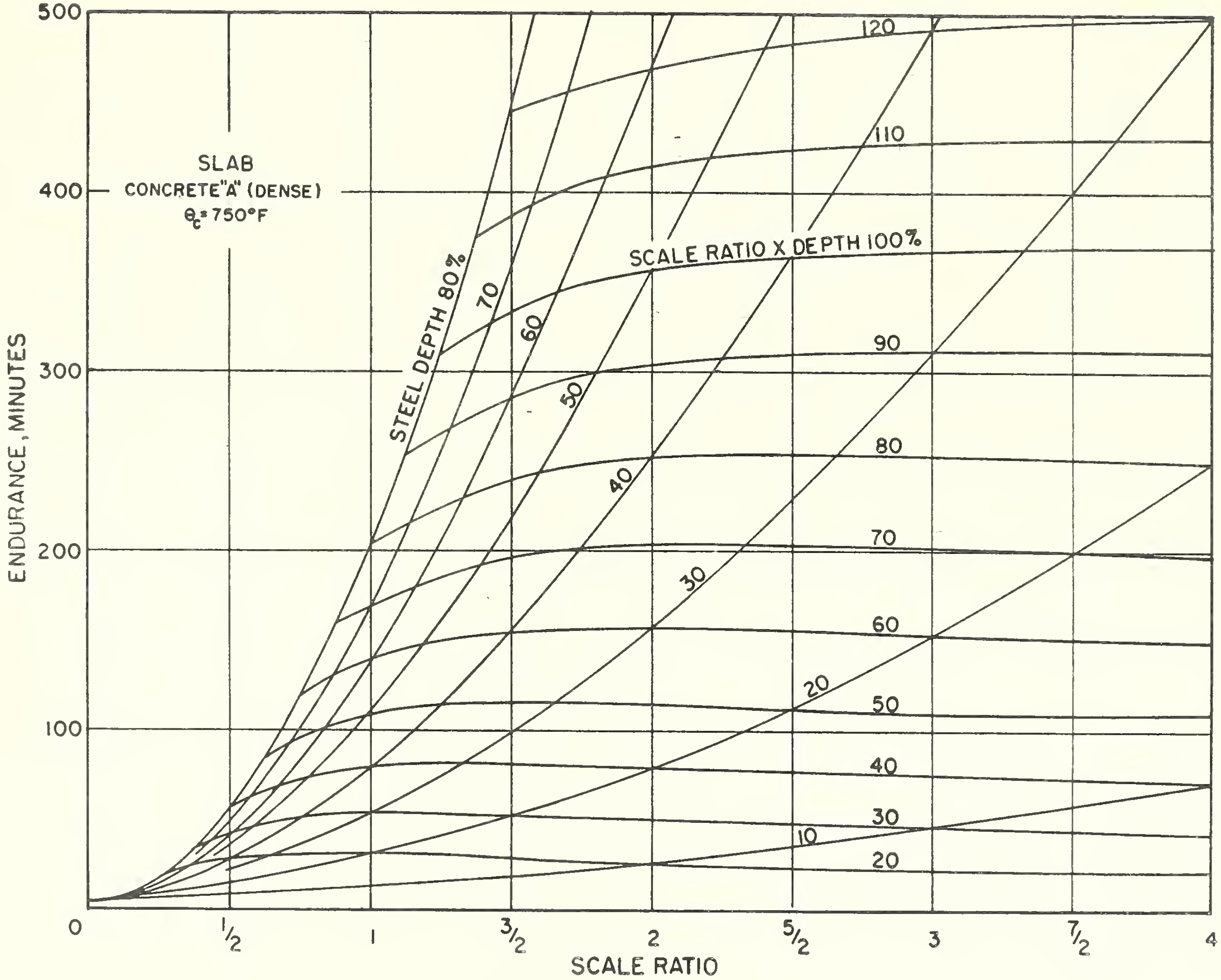
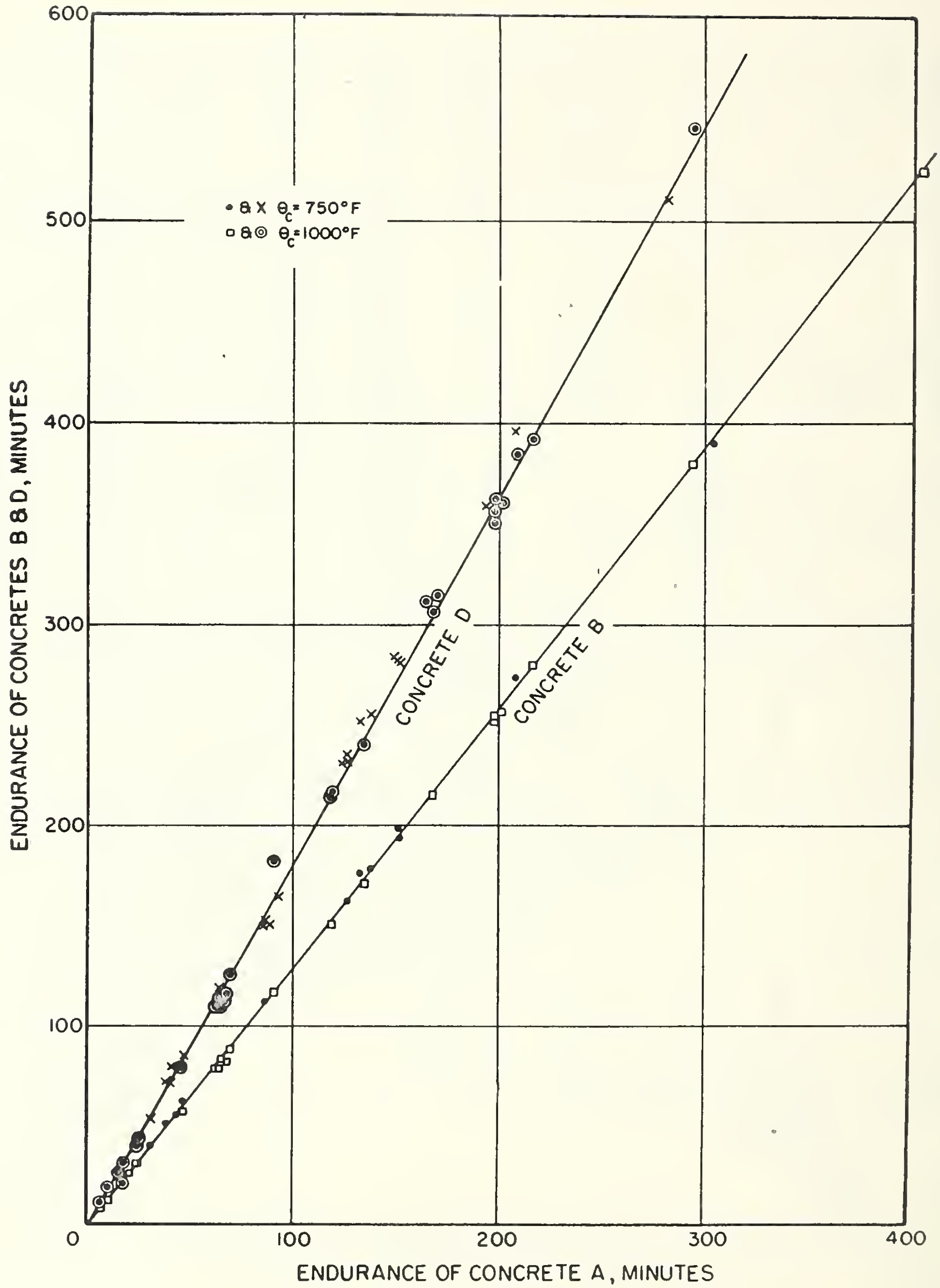
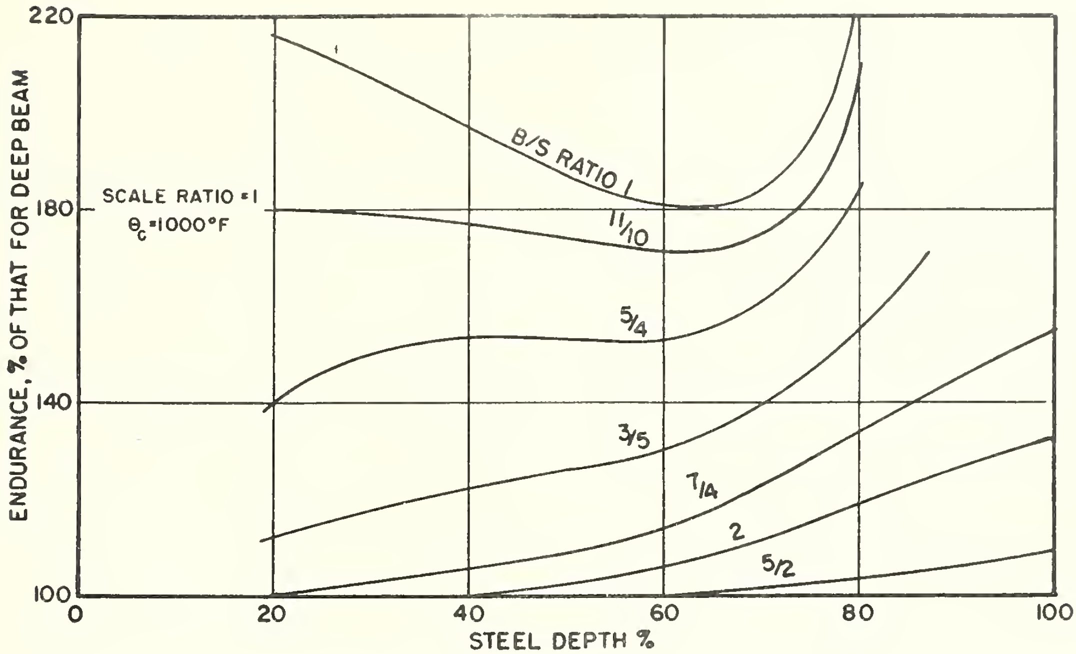

SLAB
CONCRETE "A" (DENSE)
$\theta_c = 750°F$
ENDURANCE, MINUTES
SCALE RATIO
STEEL DEPTH 80%
SCALE RATIO X DEPTH 100%
0
100
200
300
400
500
1/2
1
3/2
2
5/2
3
7/2
4
70
60
50
40
30
20
10
120
110
90
80
70
60
50
40
30
20

FIG. 11



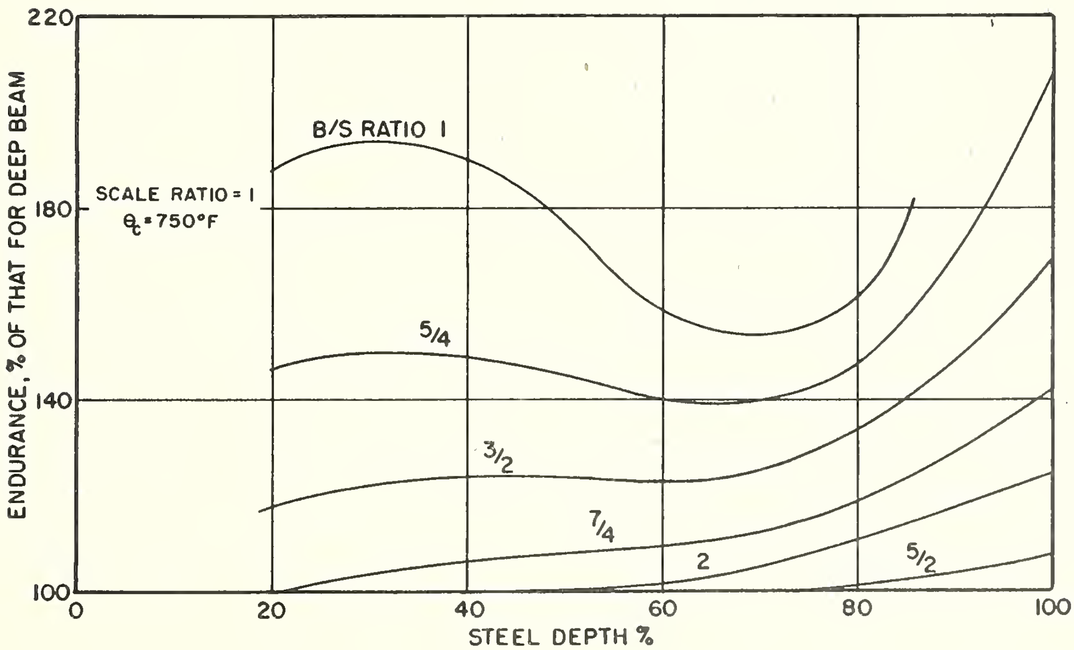
• & X θ_c = 750°F
□ & ◎ θ_c = 1000°F
CONCRETE D
CONCRETE B
ENDURANCE OF CONCRETES B & D, MINUTES
ENDURANCE OF CONCRETE A, MINUTES
0
100
200
300
400
500
600

FIG. 12



220
180
140
100
0
20
40
60
80
100
ENDURANCE, % OF THAT FOR DEEP BEAM
STEEL DEPTH %
SCALE RATIO = 1
θc = 1000°F
B/S RATIO 1
11/10
5/4
3/5
7/4
2
5/2

FIG. 13



220
180
140
100
0
20
40
60
80
100
ENDURANCE, % OF THAT FOR DEEP BEAM
STEEL DEPTH %
SCALE RATIO = 1
θc = 750°F
B/S RATIO 1
5/4
3/2
7/4
2
5/2

FIG. 14

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the front cover.

Reports and Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers and reports. Reports are issued to the sponsoring agency of a particular project or program. Published papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three monthly periodicals, available from the Government Printing Office: The Journal of Research, which presents complete papers reporting technical investigations; the Technical News Bulletin, which presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions, which provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: The Applied Mathematics Series, Circulars, Handbooks, Building Materials and Structures Reports, and Miscellaneous Publications.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.25) and its Supplement ($0.75), available from the Superintendent of Documents, Government Printing Office, Washington 25, D. C.

Inquiries regarding the Bureau's reports should be addressed to the Office of Technical Information, National Bureau of Standards, Washington 25, D. C.

NBS